

L Number	Hits	Search Text	DB	Time stamp
1	176	(708/492).CCLS.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2003/09/10 12:43
2	6	((708/492).CCLS.) and @ad<19990120 and (inver\$3).ti.	USPAT; US-PGPUB	2003/09/10 12:43
4	6	((708/491-492) or (708/620)).CCLS.) and @ad<19990120 and (inver\$3).ti.	USPAT; US-PGPUB	2003/09/10 12:43
7	2	((708/491-492) or (708/620)).CCLS.) and @ad<19990120 and (inver\$3).ti.) and ((shift adj register) "LSFB")	USPAT; US-PGPUB	2003/09/10 12:49
-	225	((@ad<19990120 and "finite field") and circuit) and arithmetic	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2003/09/04 15:59
-	47	((@ad<19990120 and "finite field") and circuit) and arithmetic) and propagat\$3	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2003/09/04 16:01
-	39	((@ad<19990120 and "finite field") and circuit) and arithmetic) and propagat\$3) and integer	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2003/09/04 16:01
-	4	((@ad<19990120 and "finite field") and circuit) and arithmetic) and (carry near propagat\$3)) and integer	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2003/09/04 16:02
-	6	((@ad<19990120 and "finite field") and circuit) and arithmetic) and (carry near propagat\$3)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2003/09/04 16:11
-	108	((380/28).CCLS.) and (galois "finite field")	USPAT; US-PGPUB	2003/09/04 16:13
-	28	"finite field" and ((without no) near carry)	USPAT; US-PGPUB	2003/09/04 16:55
-	5	"finite field" same ((without no) near carry)	USPAT; US-PGPUB	2003/09/05 09:17
-	3	"product-sum" and (finite adj field)	USPAT; US-PGPUB	2003/09/04 17:02
-	2	("product-sum" and (finite adj field)) and @ad<19990120	USPAT; US-PGPUB	2003/09/05 09:16
-	23	"elliptic" and "RSA" and coprocessor	USPAT; US-PGPUB	2003/09/05 09:57
-	702	((714/808) or (708/252, 603, 625, 653, 656)).CCLS.	USPAT; US-PGPUB	2003/09/09 09:40
-	635	(708/230, 491, 492, 654, 655).CCLS.	USPAT; US-PGPUB	2003/09/09 09:40
-	419	@ad<19990120 and iterat\$4 and (modulo modulus modular) and (multipl\$7) and \$crypt\$	USPAT; US-PGPUB	2003/09/09 10:42
-	154	@ad<19990120 and iterat\$4 and (modulo modulus modular) and (divid\$3 divis\$3) and ("finite field" galois)	USPAT; US-PGPUB	2003/09/09 12:41
-	24	@ad<19990120 and ((modulo modulus modular remainder) same (divid\$3 divis\$3)) and (iterat\$4 near subtract\$3)	USPAT; US-PGPUB	2003/09/09 14:47
-	46	@ad<19990120 and (inverse same multipl\$7 same divi\$4) and ("finite field" galois) and (modulo)	USPAT; US-PGPUB	2003/09/09 15:02
-	2	@ad<19990120 and (inverse same multipl\$7 same divi\$4 same (instead rather)) and ("finite field" galois) and (modulo)	USPAT; US-PGPUB	2003/09/09 15:00
-	27	@ad<19990120 and (multipl\$7 near divi\$4 near inver\$4)	USPAT; US-PGPUB	2003/09/09 16:00

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-	39	((@ad<19990120 and ((multipl\$7 near inver\$4) same divi\$4)) not (@ad<19990120 and (multipl\$7 near divi\$4 near inver\$4))) and @ad<19990120) and ("finite field" galois)	USPAT; US-PGPUB	2003/09/09 16:10
-	17	(multiplicative adj inverse).ti.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/09 16:29
-	4	("4473887" "4567568" "4574361" "4800515").PN.	USPAT	2003/09/09 16:18
-	28	@ad<19990120 and (inver\$3 and ("finite field" Galois)).ti.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/09 16:38
-	176	(708/492).CCLS.	USPAT; US-PGPUB	2003/09/09 17:20
-	2	(@ad<19990120 and (inverse same multipl\$7 same divi\$4 same (instead rather)) and ("finite field" galois) and (modulo)) and inverse	USPAT; US-PGPUB	2003/09/09 17:21
-	65	(@ad<19990120 and "multiplicative inverse") and (Galois "finite field")	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/10 08:41
-	48	((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/10 08:41
-	46	(((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and bit	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/10 08:41
-	25	(((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and bit) and significant	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/10 09:22
-	25	(((((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and bit) and significant) and inver\$4	USPAT; US-PGPUB	2003/09/10 09:22
-	26	(((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and significant	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB USPAT	2003/09/10 09:22
-	25	(((((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and significant) and inver\$4	USPAT; US-PGPUB	2003/09/10 11:00
-	66	(@ad<19990120 and inver\$4 and (linear adj feedback adj shift adj register)) and galois	USPAT; US-PGPUB	2003/09/10 11:07
-	59	((@ad<19990120 and inver\$4 and (linear adj feedback adj shift adj register)) and galois) not ((((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and significant) and inver\$4	USPAT; US-PGPUB	2003/09/10 11:09
-	4	(((@ad<19990120 and inver\$4 and (linear adj feedback adj shift adj register)) and galois) not ((((@ad<19990120 and "multiplicative inverse") and (Galois "finite field")) and shift\$3) and significant) and inver\$4)) and (multiplicative adj inverse)	USPAT; US-PGPUB	2003/09/10 13:55



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ABSTRACT

The paradigm of algorithm analysis has achieved major pre-eminence in the field of symbolic and algebraic manipulation in the last few years. A major factor in its success has been the use of modular arithmetic. Application of this technique has proved effective in reducing computing times for algorithms covering a wide variety of symbolic mathematical problems. This paper is intended to review the basic theory underlying modular arithmetic. In addition, attention will be paid to certain practical problems which arise in the construction of a modular arithmetic system. A second area of importance in symbol manipulation is the theory of finite fields. A recent algorithm for polynomial factorization over a finite field has led to faster algorithms for factorization over the field of rationals. Moreover, the work in modular arithmetic often consists of manipulating elements in a finite field. Hence, this paper will outline some of the major theorems for finite fields, hoping to provide a basis from which an easier grasp of these new algorithms can be made.

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✦ INDEX TERMS

Primary Classification:

F. Theory of Computation

✦ F.2 ANALYSIS OF ALGORITHMS AND PROBLEM COMPLEXITY

✦ F.2.1 Numerical Algorithms and Problems

✦ **Subjects:** Computations in finite fields

Additional Classification:

F. Theory of Computation

✦ F.2 ANALYSIS OF ALGORITHMS AND PROBLEM COMPLEXITY

✦ F.2.1 Numerical Algorithms and Problems

✦ **Subjects:** Number-theoretic computations (e.g., factoring, primality testing)

I. Computing Methodologies

✦ I.1 SYMBOLIC AND ALGEBRAIC MANIPULATION

✦ I.1.0 General

General Terms:

Algorithms, Theory

Keywords:

Exact multiplication, Finite fields, Modular arithmetic, Symbol manipulation;

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